

Axion Decay of a Photon in an External Electromagnetic Field

N.V. Mikheev, A.Ya. Parkhomenko and L.A. Vassilevskaya
*Yaroslavl (Demidov) State University,
 Sovietskaya 14, Yaroslavl 150000, Russia*

Abstract

An interaction of a pseudoscalar particle with two photons induced by an external electromagnetic field is used to study the photon decay $\gamma \rightarrow \gamma a$ where a is a pseudoscalar particle associated with the Peccei-Quinn $U(1)$ symmetry. The field-induced axion emission by photon is analyzed as a possible source of energy losses by astrophysical objects.

1 Introduction

The physical nature of dark matter in the universe still remains an unresolved mystery. Axions [1, 2] which appear as Nambu-Goldstone bosons of the spontaneously broken Peccei-Quinn symmetry $U_{PQ}(1)$ are one of the well-motivated candidates for the cold dark matter [3, 4, 5]. In analogy to neutral pions, axions generically interact with photons according to Lagrangian:

$$\mathcal{L}_{a\gamma} = -\frac{g_{a\gamma}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} a = g_{a\gamma} \mathbf{E} \cdot \mathbf{B} a \quad (1)$$

with a strength

$$g_{a\gamma} = \frac{\alpha}{\pi f_a} \xi, \quad (2)$$

where f_a is the energy scale of the symmetry breaking and ξ is a model-dependent factor of order unity, $F_{\mu\nu}$ is the electromagnetic field strength tensor, $\tilde{F}_{\mu\nu}$ its dual, and \mathbf{E} and \mathbf{B} are the electric and magnetic fields, respectively; a is the axion field. Existing axion models also contain an interaction of axions with charged fermions (usual or exotic):

$$\mathcal{L}_{af} = \frac{g_{af}}{2m_f} (\bar{f} \gamma_\mu \gamma_5 f) \partial_\mu a, \quad (3)$$

which automatically leads to an electromagnetic coupling of the form in Eq. (1) because of the triangle loop $a\gamma\gamma$ -amplitude [3]. Here $g_{af} = C_f m_f / f_a$ is a dimensionless coupling constant, C_f is a model-dependent factor, f is the fermion field with mass m_f ; γ_μ, γ_5 are Dirac γ -matrices.

The two-photon-axion interaction vertex allows for the axion radiative decay $a \rightarrow \gamma\gamma$ [3], for the Primakoff conversion $a \leftrightarrow \gamma$ in the presence of electric or magnetic fields [6] as well as for plasmon decay $\gamma_T \rightarrow \gamma_L a$ [7, 8] and coalescence $\gamma_L \gamma_T \rightarrow a$ [8]. The last two processes are kinematically possible because of the dispersion relations of electromagnetic excitations in a plasma which differ significantly from the vacuum dispersion.

It is known that an external electromagnetic field plays the role of an active medium in its influence on particle properties. Due to a nontrivial kinematics of charged particles and a photon dispersion in the external field, such axion processes as the axion decay into electron-positron pair $a \rightarrow e^+ e^-$ [9, 10], the axion cyclotron emission $e \rightarrow ea$ [11, 12] are not only opened kinematically but also become substantial.

Due to a very weak interaction of axions with a matter (the latest astrophysical data yield $f_a \gtrsim 10^{10}$ GeV [3]), the processes with axion emission could be of great importance in astrophysics as an additional source of star energy losses¹. In the studies of processes occurring inside astrophysical objects, one has to take into account the influence of both components of the active medium – a plasma and a magnetic field. A situation is also possible when the field component dominates; for example, in a supernova explosion or in a coalescence of neutron stars, a region outside the neutrino sphere of order of hundred kilometers with a strong magnetic field and a rather rarefied plasma could exist.

In this letter we investigate a forbidden in vacuum axion decay of photon $\gamma \rightarrow \gamma a$ in an external electromagnetic field using the effective $a\gamma\gamma$ -interaction [13] and estimate a possible influence of this decay on the star cooling process.

2 Field-Induced “Effective Masses” of the Particles

In calculating $\gamma \rightarrow \gamma a$ decay in the external field, we have to integrate over the phase space of the final particles (photon, axion) taking into account their non-trivial kinematics. The kinematics depend substantially on the particles dispersion relations in the electromagnetic field which plays the role of an anisotropic medium. The photon “effective masses” squared μ_λ^2 induced by the external field are defined as the eigenvalues of the photon polarization operator $\Pi_{\mu\nu}$:

$$\Pi_{\mu\nu} = i \sum_{\lambda=1}^3 \Pi^{(\lambda)} \varepsilon_\mu^{(\lambda)} \varepsilon_\nu^{(\lambda)}, \quad (4)$$

¹For example, the upper bound on m_a is obtained from the requirement that stars not lose too much energy by axions.

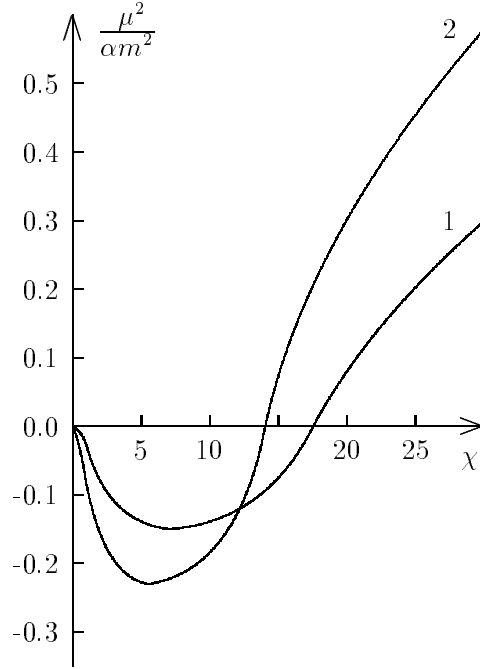


Figure 1: The photon dispersion curves in the crossed field.

where $\varepsilon_\mu^{(\lambda)}$ are the photon polarization vectors. Note that eigenmodes of two transverse photons with polarization vectors $\varepsilon_\mu^{(1)}$ and $\varepsilon_\mu^{(2)}$:

$$\varepsilon_\mu^{(1)} = \frac{(qF)_\mu}{\sqrt{(qFFq)}}, \quad \varepsilon_\mu^{(2)} = \frac{(q\tilde{F})_\mu}{\sqrt{(qFFq)}}, \quad (5)$$

are realized in the electromagnetic field. Here $F_{\mu\nu}$ is the external field tensor and q_μ is the photon four-momentum.

Note that an arbitrary weak smooth field in which a relativistic particle propagates is well-described by the constant crossed field limit ($\mathbf{E} \perp \mathbf{B}$, $E = B$). Thus, calculations in this field possess a great extent of generality and acquire interest by themselves. Below we will consider the constant crossed field as the external field. In this case the dynamic parameter χ is the only field invariant.

The analysis of $\Pi_{\mu\nu}$ in one-loop approximation in the crossed field [14] shows that the dispersion curves (Fig. 1) corresponding to the above-mentioned photon eigenmodes (5), though being similar in their qualitative behavior, are quantitatively different. The difference in values of the field-induced “effective masses” squared of the first- and second-photon eigenmodes makes the process of the photon splitting $\gamma \rightarrow \gamma a$ possible, where a is an arbitrary relatively light pseudoscalar with mass m_a .

The field-induced contribution to the small axion mass ² δm_a (the real part of $a \rightarrow$

²The allowed range for the axion mass is strongly constrained by astrophysical and cosmological considerations, as a result $10^{-5} \text{ eV} \lesssim m_a \lesssim 10^{-2} \text{ eV}$ [3, 4, 5].

$f\tilde{f} \rightarrow a$ transition amplitude via the fermion loop) is negligible:

$$\frac{\delta m_a^2}{m_a^2} \lesssim 10^{-10} C_e^2 \chi^{2/3}, \quad (6)$$

where C_e is the model-dependent factor which determines the axion-electron coupling constant g_{ae} . Hereafter we will take axion as a massless particle.

3 Matrix Element

The effective axion-photon vertex was obtained in Ref. [13] and used to analyze the axion radiative decay $a \rightarrow \gamma\gamma$ [15]. As in the case of $a \rightarrow \gamma\gamma$, the main contribution to the amplitude of the photon decay $\gamma \rightarrow \gamma a$ comes from the bilinear on the external field terms of $a\gamma\gamma$ -vertex and can be presented in the form:

$$M \simeq \frac{\alpha}{\pi} \sum_f \frac{Q_f^2 g_{af}}{m_f} \left\{ \frac{\chi}{\chi_2} (f_1 \tilde{\mathcal{F}})(f_2 \mathcal{F}) J(\chi_1, \chi_2) - \frac{\chi}{\chi_1} (f_2 \tilde{\mathcal{F}})(f_1 \mathcal{F}) J(\chi_2, \chi_1) \right\}, \quad (7)$$

$$\begin{aligned} \mathcal{F}_{\mu\nu} &= \frac{F_{\mu\nu}}{B_f}, \quad \tilde{\mathcal{F}}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} \mathcal{F}_{\alpha\beta}, \quad B_f = \frac{m_f^2}{eQ_f}, \\ (f_i)_{\alpha\beta} &= q_{i\alpha} \varepsilon_{i\beta} - q_{i\beta} \varepsilon_{i\alpha}, \quad (\tilde{f}_i)_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\mu\nu} (f_i)_{\mu\nu}, \quad i = 1, 2, \end{aligned}$$

$$\begin{aligned} J(\chi_1, \chi_2) &= \int_0^1 dx \int_0^{1-x} dy y (1-y-2x) \eta^3 (1-\eta f(\eta)), \\ f(\eta) &= i \int_0^\infty du \exp \left[-i \left(\eta u + \frac{1}{3} u^3 \right) \right], \\ \eta(\chi_1, \chi_2) &= \left[(x(1-x)\chi_2 + y(1-y)\chi_1)^2 \right. \\ &\quad \left. - 4xy(xy + (1-x-y)(x+y)) \chi_1 \chi_2 \right]^{-1/3}, \end{aligned} \quad (8)$$

where $f(\eta)$ is the Hardy-Stokes function, χ , χ_1 and χ_2 are the dynamic parameters

$$\chi^2 = \frac{(p\mathcal{F}\mathcal{F}p)}{m_f^2}, \quad \chi_i^2 = \frac{(q_i\mathcal{F}\mathcal{F}q_i)}{m_f^2}, \quad i = 1, 2. \quad (9)$$

Here q_1 and q_2 are the four-momenta of the initial and final photons, respectively; p is the axion four-momentum. The summation in Eq. (7) is over the virtual fermions f with mass m_f and the relative electric charge Q_f ; $e > 0$ is the elementary charge; B_f is the critical value of the magnetic field strength for the fermion. We have neglected terms of order m_a^2/E_a^2 , μ_i^2/E_a^2 in the argument of the function $f(\eta)$.

The amplitude, Eq. (7), is significantly simplified in the case of small values of the dynamic parameters. In this case the decay of the photon with the first polarization (see Fig. 1) is allowed kinematically due to the condition $\mu_1^2 > \mu_2^2$:

$$\gamma^{(1)} \rightarrow \gamma^{(2)} + a. \quad (10)$$

With the photon polarization vectors, Eq. (5), the amplitude is:

$$M \simeq -\frac{4\alpha}{\pi} t (1-t) \sum_f Q_f^2 g_{af} m_f \chi_1^2 J(t\chi_1, \chi_1), \quad (11)$$

where $t = \omega_2/\omega_1$ is the relative energy of the final photon. The function J defined in (8) has the following asymptotic behavior at small values of its arguments:

$$J(t\chi_1, \chi_1) \Big|_{\chi_1 \ll 1} \simeq \frac{2}{63} \chi_1^2 (1-2t) + O(\chi_1^4).$$

The limit of large values of the dynamic parameter can only be realized in the central regions of astrophysical objects with the strong magnetic field where the characteristic temperatures $T \gtrsim 10 \text{ MeV}$. But in these regions from both components of the active medium, the plasma and the magnetic field, the plasma component dominates. Thus Eq. (7) cannot be used to estimate possible astrophysical applications because it does not take into account the plasma influence.

4 Axion Emissivity

The probability of the photon decay in the limit of small values of dynamic parameter χ_1 can be presented in the form:

$$W^{(F)} = \frac{1}{16\pi\omega_1} \int_0^1 dt |M|^2 \simeq 4.8 \cdot 10^{-6} \left(\frac{\alpha}{\pi}\right)^2 \frac{(\sum_f Q_f^2 g_{af} m_f \chi_1^4)^2}{\pi\omega_1}. \quad (12)$$

To illustrate a possible application of the result obtained, we calculate the contribution of this process to the axion emissivity Q_a , i.e. the rate of energy loss per unit volume, of the photon gas:

$$Q_a = \int \frac{d^3 q_1}{(2\pi)^3} \omega_1 n_B(\omega_1) \int_0^1 dt \frac{dW^{(F)}}{dt} (1-t) (1+n_B(\omega_1 t)), \quad (13)$$

where $n_B(\omega_1)$ and $n_B(\omega_1 t)$ are the Planck distribution functions of the initial and final photons at temperature T , respectively. In Eq. (13) we have taken into account that the photon of only one polarization (the first one in our case) splits. Note that the dynamic

parameter χ_1 in $dW^{(F)}/dt$ depends on the photon energy ω_1 and the angle θ between the initial photon momentum \mathbf{q}_1 and the magnetic field strength \mathbf{B} :

$$\chi_1 = \frac{\omega_1}{m_f} \frac{B}{B_f} \sin \theta.$$

Neglecting the photon “effective masses” squared ($\omega_1^2 = |\mathbf{q}_1|^2 + \mu_1^2 \simeq |\mathbf{q}_1|^2$), the result of the calculation of the axion emissivity Q_a (13) becomes:

$$Q_a \simeq 2.15 \frac{\alpha^2}{\pi^5} \left(\sum_f \frac{Q_f^2 g_{af}}{m_f^3 B_f^4} \right)^2 T^{11} B^8. \quad (14)$$

Below we estimate the contribution of the photon decay $\gamma \rightarrow \gamma a$ into the axion luminosity L_a in a supernova explosion from a region of order of hundred kilometers in size outside the neutrino sphere. In this region a rather rarefied plasma with the temperature of order of MeV and the magnetic field of order 10^{13} G can exist. Under these conditions the estimation of the axion luminosity is:

$$L_a \simeq 3 \times 10^{33} \frac{\text{erg}}{\text{s}} \left(\frac{g_{ae}}{10^{-13}} \right)^2 \left(\frac{T}{1 \text{ MeV}} \right)^{11} \left(\frac{B}{10^{13} \text{ G}} \right)^8 \left(\frac{R}{100 \text{ km}} \right)^3. \quad (15)$$

The comparison of Eq. (15) with the total neutrino luminosity $L_\nu \sim 10^{52}$ erg/s from the neutrino sphere shows that the contribution of the photon decay to the supernova energy losses is very small and does not allow to get a new restriction on the axion-electron coupling g_{ae} .

5 Conclusions

In this letter we have studied the field-induced photon decay $\gamma \rightarrow \gamma a$ (a is a light pseudoscalar particle). For the pseudoscalar particle, we considered the most widely discussed particle, the axion, corresponding to the spontaneous breaking of the Peccei-Quinn symmetry. This is forbidden in vacuum process but it becomes kinematically possible because photons of different polarizations obtain different field-induced “effective masses” squared. At the same time an external field influence on the axion mass is negligible.

The process $\gamma \rightarrow \gamma a$ could be of interest as an additional source of energy losses by astrophysical objects. We considered the case of small values of the dynamic parameter which can be realized, for example, in a supernova explosion. While this process and its evaluation are conceptually quite intriguing, the actual energy-loss rate appeared to be rather small in comparison with the neutrino luminosity in the conditions considered.

Acknowledgments

This work was partially supported by INTAS under grant No. 96-0659 and by the Russian Foundation for Basic Research under grant No. 98-02-16694. The work of N.V. Mikheev

was supported under grant No. d98-181 by International Soros Science Education Program.

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